Aristotle University of Thessaloniki

Academic Open Courses

Groundwater Hydraulics

Unit 2: Groundwater flows towards ditches and wells

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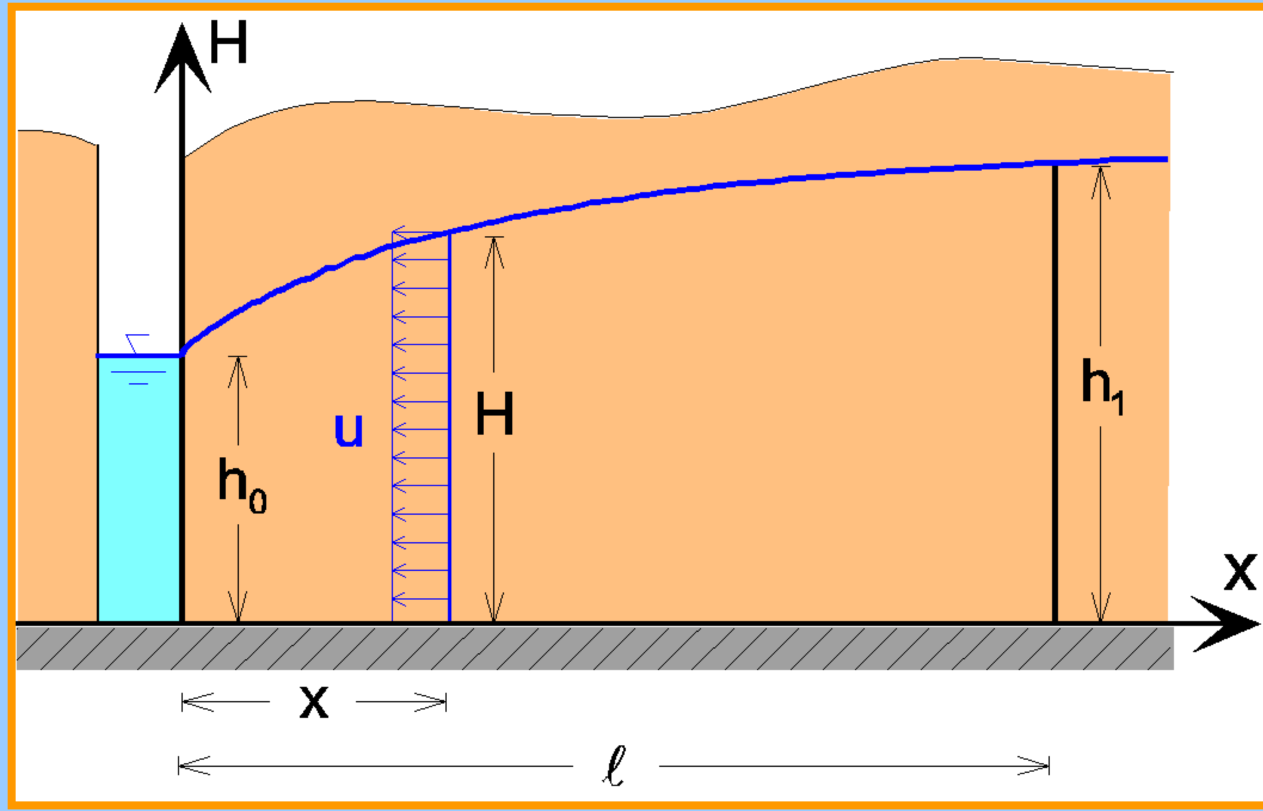
Steady groundwater flows

One dimensional flow-Flow towards a ditch

a) Free surface flow

$$q = K\frac{h_1^2 - h_0^2}{2l}$$

$$H^2 = h_0^2 + \frac{2qx}{K}$$



Where

q is the flow rate per meter of length of the ditch (m^2/s) and

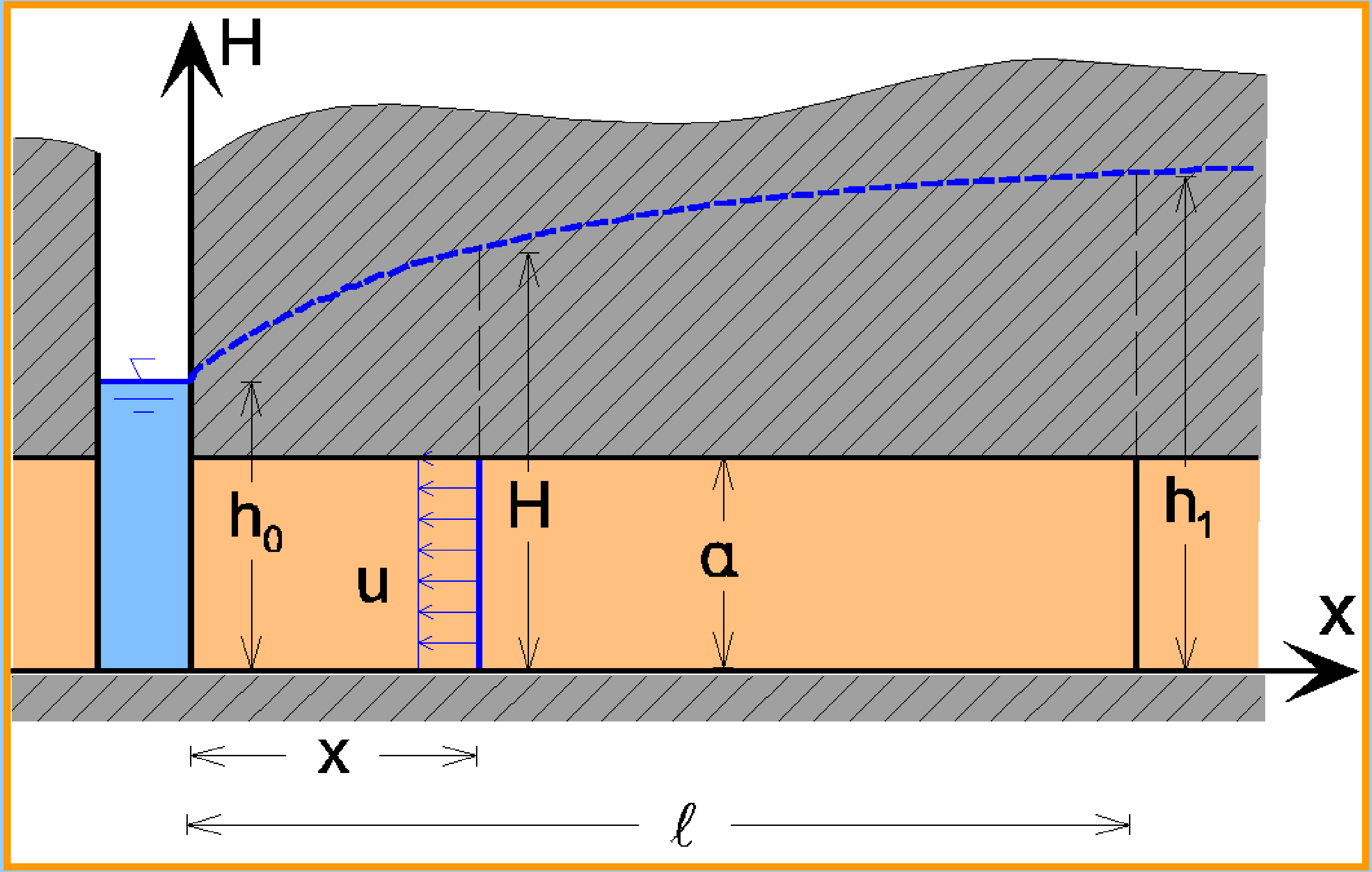
K is the hydraulic conductivity.

Source of figure: https://opencourses.auth.gr/modules/units/?course=OCRS466&id=4825

One dimensional flow-Flow towards a ditch

b) Confined flow

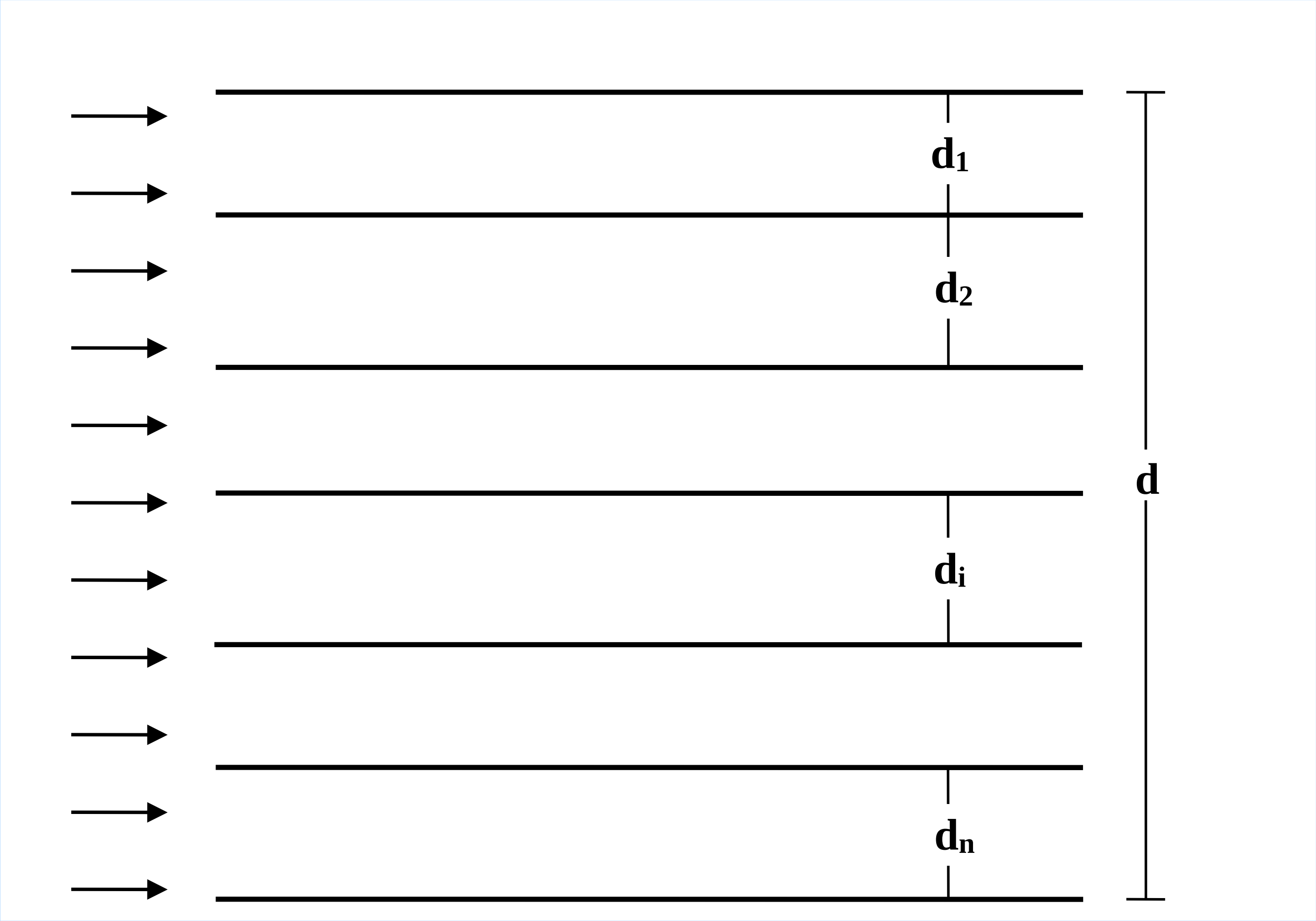
$$q = \frac{K\alpha}{l}\left(h_1 - h_0\right)$$



Source of figure: https://opencourses.auth.gr/modules/units/?course=OCRS466&id=4825

Flow through stratified flow media

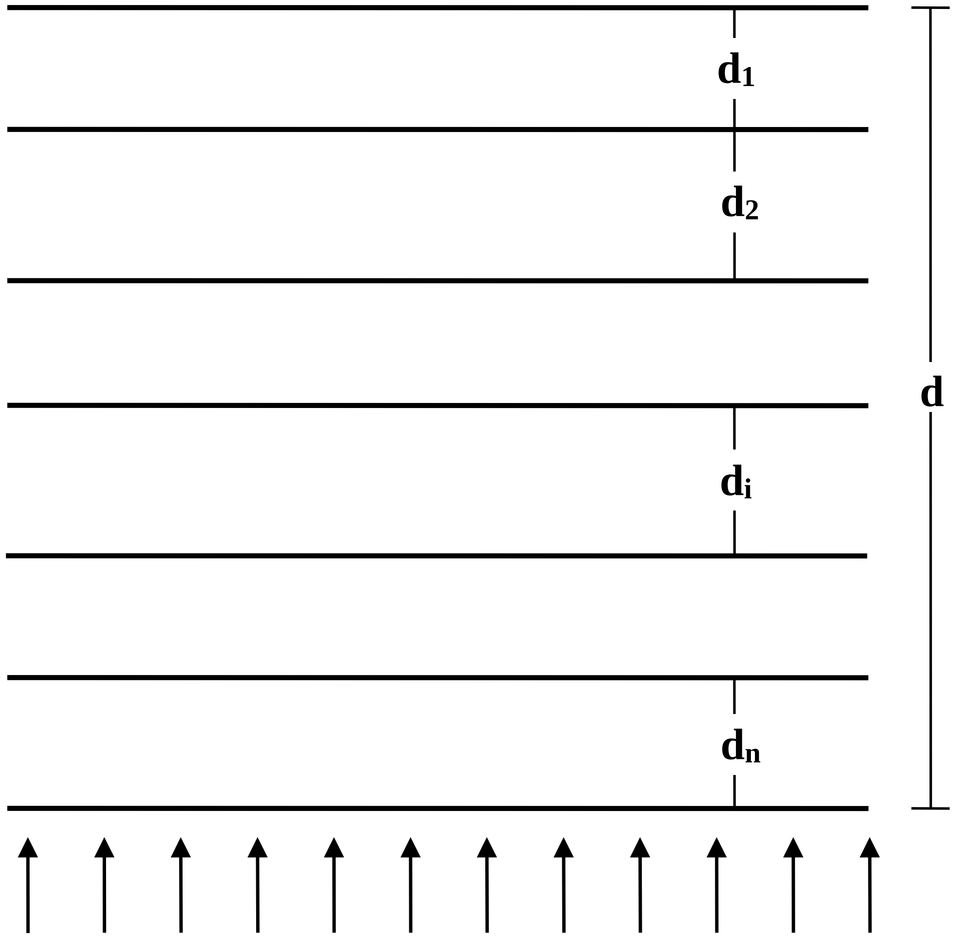
a) Ground water flow parallel to aquifer layers



Equivalent hydraulic conductivity coefficient

$$K_{eq1} = \sum_{i=1}^{n} \frac{K_i d_i}{d}$$

b) Ground water flow perpendicular to aquifer layers



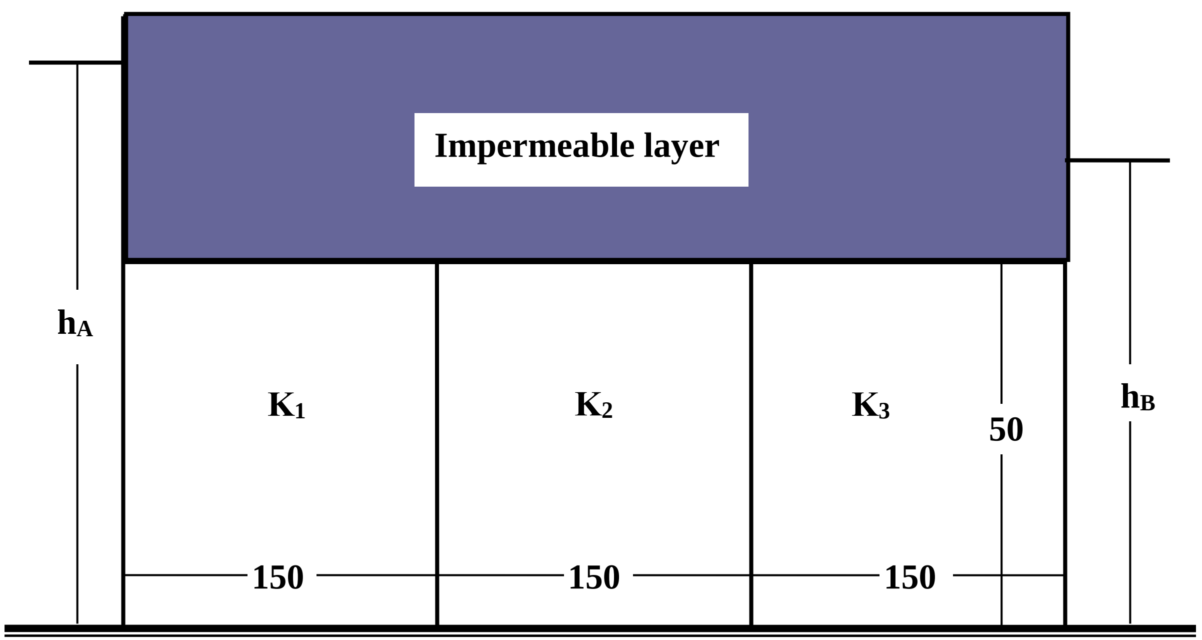
Equivalent hydraulic conductivity coefficient

$$K_{eq2} = \frac{d}{\sum_{i=1}^{n} \frac{d_i}{K_i}}$$

Exercise 1: Ditches A and B are hydraulically connected through the inhomogeneous aquifer that is shown in the figure. The hydraulic head value at the ditches is h_A = 80 m and h_B = 60 m respectively. The dimensions of the aquifer's layers are shown in the figure (which is drawn out of scale), while their hydraulic conductivities are equal to K_1 = 40 m/day, K_2 = 20 m/day, K_3 = 30 m/day, respectively.

a) Calculate the flow rate per meter (in m^2/day), from ditch A to ditch B.

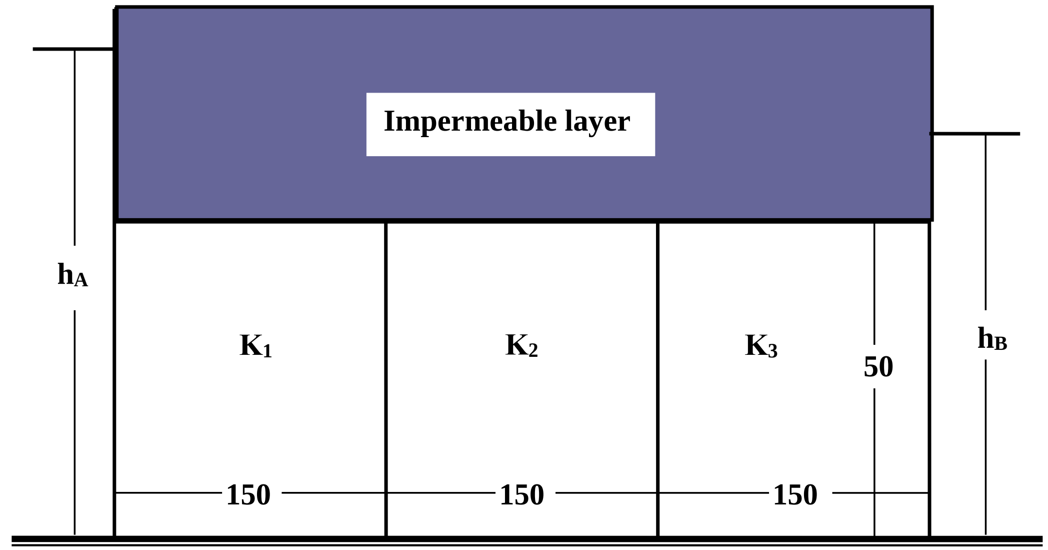
b) The hydraulic head at the middle of the distance between ditches A and B is equal, smaller or larger than 70 m? Explain your answer.



Solution

The flow is perpendicular to the layers, so:

$$K_{eq} = \frac{d}{\sum_{i=1}^{n} \frac{d_i}{K_i}}$$



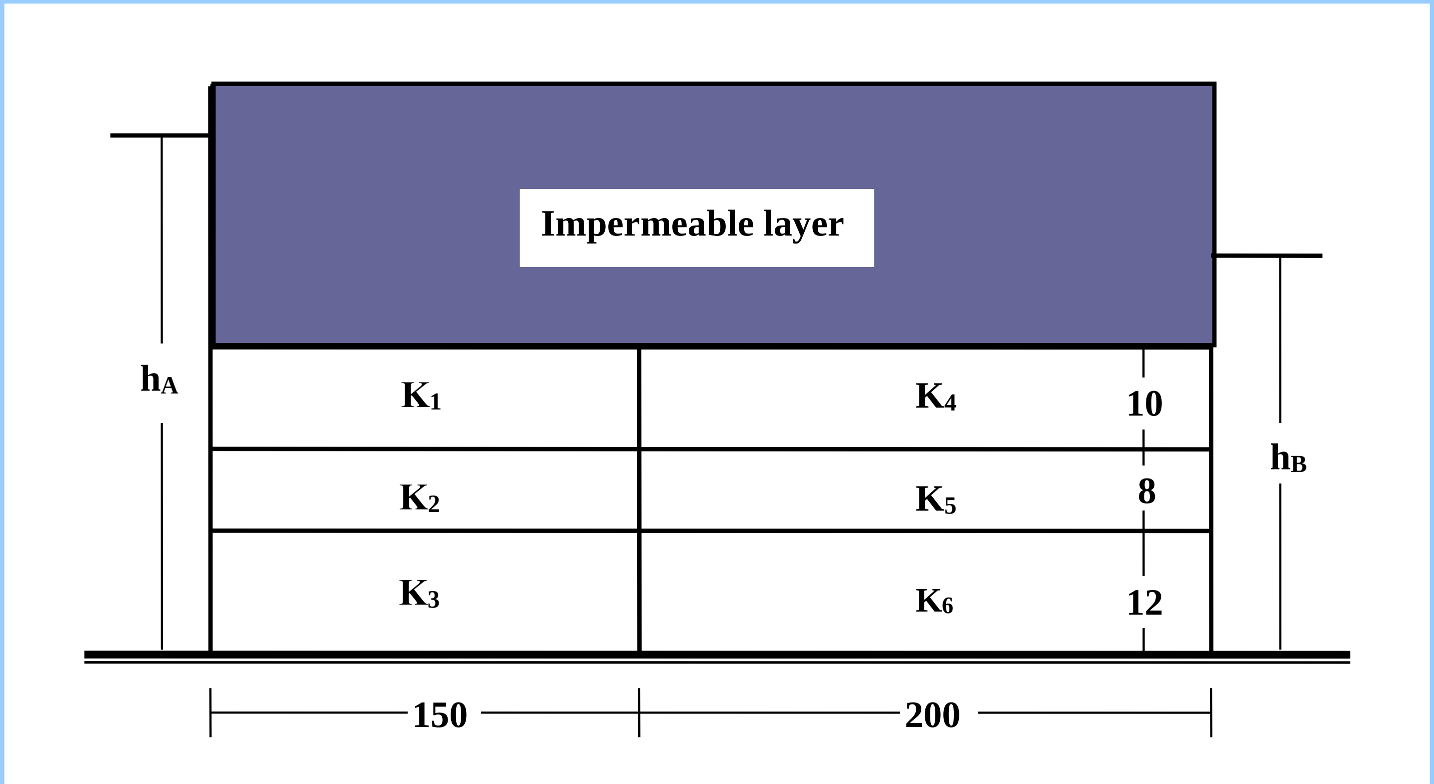
$$K_{eq} = \frac{450}{\frac{150}{40} + \frac{150}{20} + \frac{150}{30}} = \frac{450}{16.25} = 27.69 \ \ m/day$$

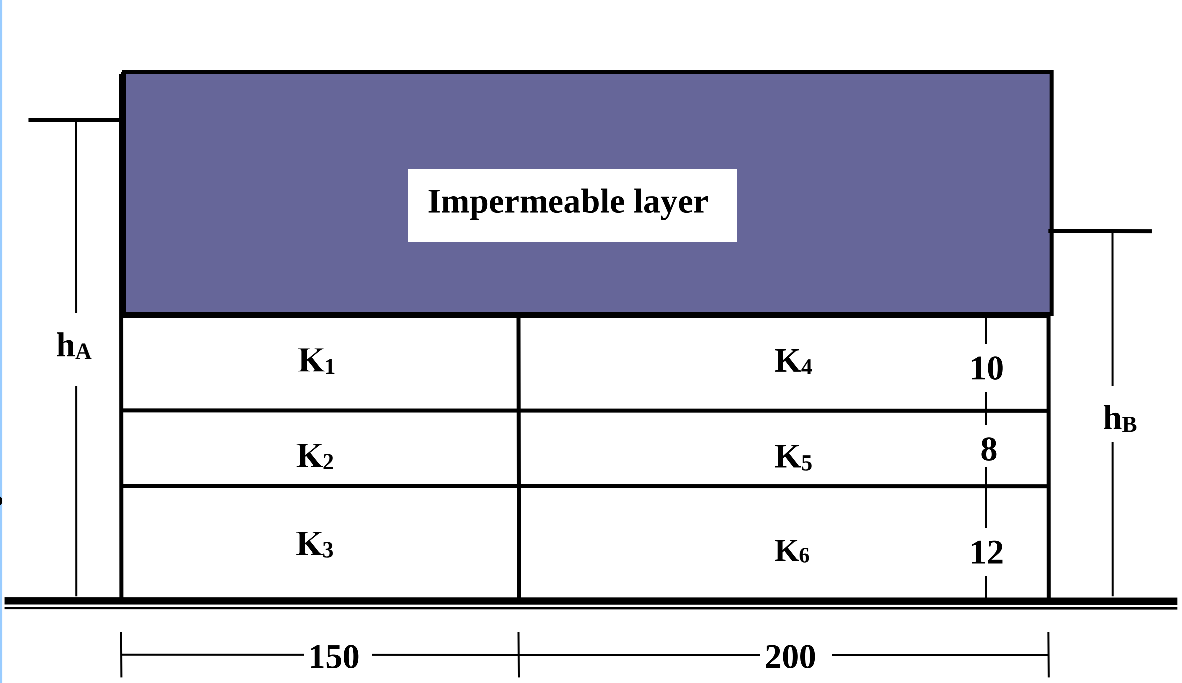
The flow rate per meter is given as:

$$q = \frac{27.69 \cdot 50}{450}(80 - 60) = 61.53 \ \ m^2/day$$

Exercise 2

Ditches A and B are hydraulically connected through an inhomogeneous aquifer, as shown in the figure. The hydraulic head value at the ditches is h_A = 60 m and h_B = 40 m respectively. The dimensions of the aquifer's layers are shown in the figure (which is drawn out of scale), while their hydraulic conductivities are equal to K_1 = 30 m/day, K_2 = 20 m/day, K_3 = 10 m/day, K_4 = 12 m/day, K_5 = 15 m/day, K_6 = 25 m/day, respectively. Calculate the flow rate per meter (in m^2/day), from ditch A to ditch B.





Solution

First option

Consider the layers 1, 2, 3. The flow is parallel to the layers. Then:

$$K_{eq123} = (30\cdot 10 + 20\cdot 8 + 10\cdot 12)/30 = 19.33 \text{ m/day}$$

Consider the layers 4, 5, 6. The flow is parallel to the layers. Then:

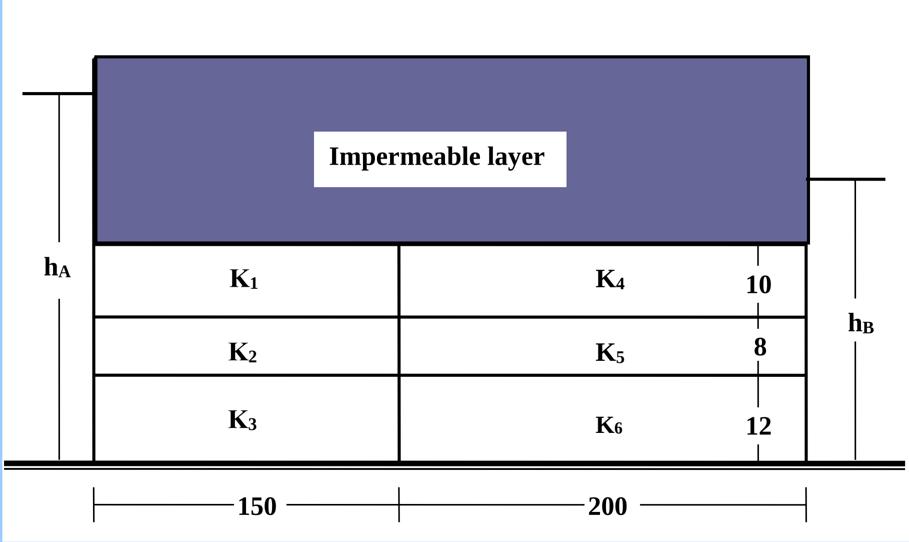
$$K_{eq456} = (12\cdot 10 + 15\cdot 8 + 25\cdot 12)/30 = 18.0 \text{ m/day}$$

Flow is perpendicular to the 2 equivalent layers, so:

$$K_{eqT} = \frac{d}{\sum_{i=1}^{2} \frac{d_i}{K_i}} = \frac{150+200}{\frac{150}{19.33}+\frac{200}{18}} = \frac{350}{18.87} = 18.55 \text{ m/day}$$

The flow rate per meter is given as:

$$q = \frac{18.55 \cdot 30}{350}(60 - 40) = 31.8 \ \ m^2 / day$$



Second option

Consider the layers 1 and 4. The flow is perpendicular to the layers. Then:

$$K_{eq14} = \frac{d}{\sum_{i=1}^{2} \frac{d_i}{K_i}} = \frac{150 + 200}{\frac{150}{30} + \frac{200}{12}} = \frac{350}{21.67} = 16.15 \ \ m / day$$

Consider the layers 2 and 5. The flow is perpendicular to the layers. Then:

$$K_{eq25} = \frac{d}{\sum_{i=1}^{2} \frac{d_i}{K_i}} = \frac{150 + 200}{\frac{150}{20} + \frac{200}{15}} = \frac{350}{20.83} = 16.8 \ \ m / day$$

Similarly, for layers 3 and 6:

$$K_{eq36} = \frac{d}{\sum_{i=1}^{2} \frac{d_i}{K_i}} = \frac{150+200}{\frac{150}{10}+\frac{200}{25}} = \frac{350}{23} = 15.22 \;\; m/day$$

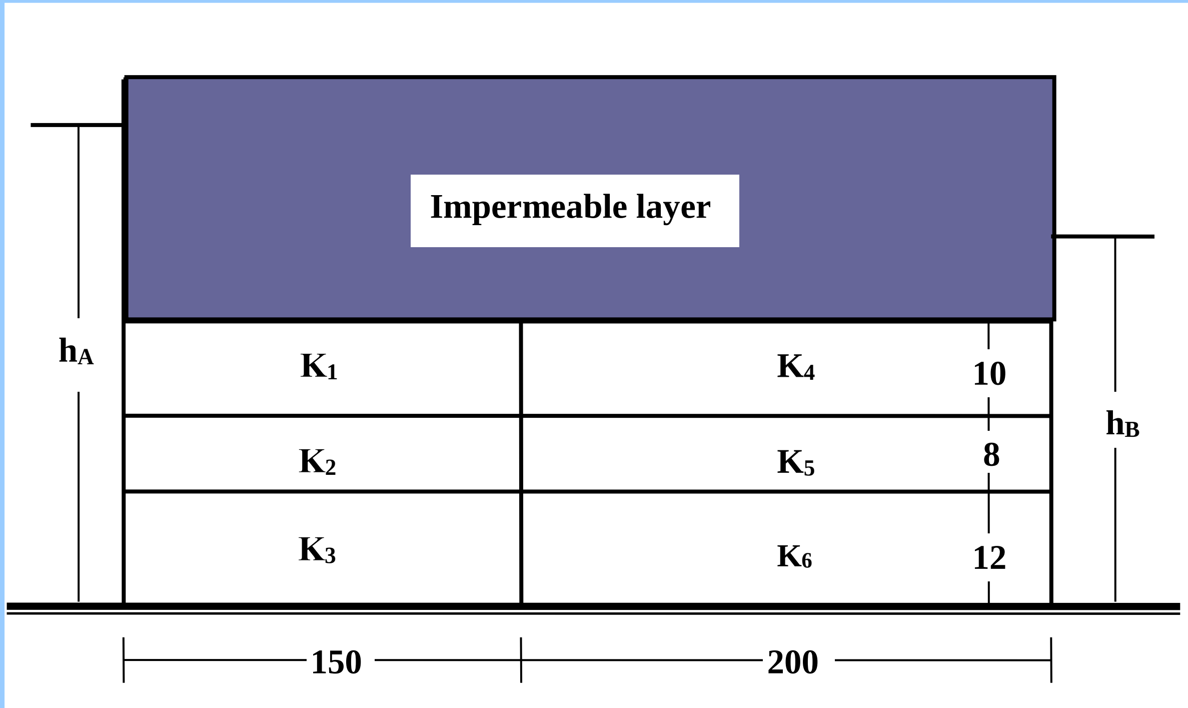
Flow is parallel to the 3 equivalent layers, so:

$$K_{eqT} = (16.15\cdot 10 + 16.8\cdot 8 + 15.22\cdot 12)/30 = 15.95 \text{ m/day}$$

Short discussion:

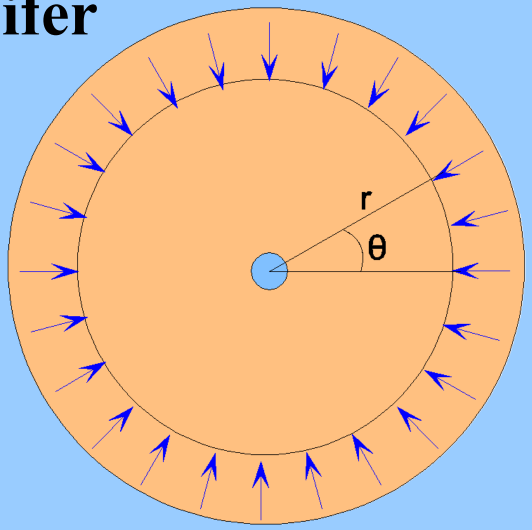
What is the reason of the discrepancy?

Which result is correct?



Flow to wells

Confined flow towards a well in an "infinite" aquifer



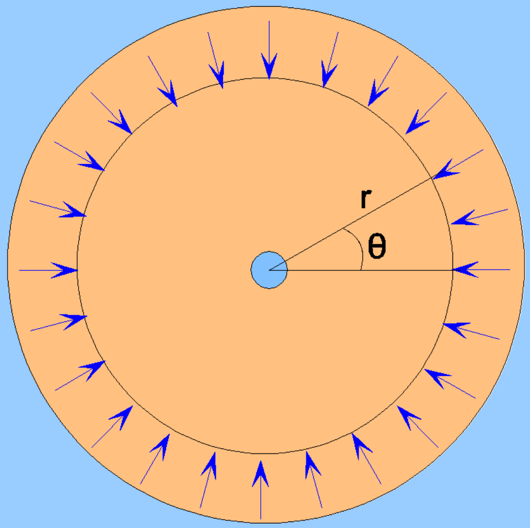
$$s = -\frac{Q}{2\pi K\alpha}\ln\frac{r}{R}$$

where s is the hydraulic head level drawdown at a distance r ftom the well, α the aquifer width, K the hydraulic conductivity and R the radius of influence.
R is a large distance from the well, where hydraulic head level drawdown is practically zero.

Source of figure: https://opencourses.auth.gr/modules/units/?course=OCRS466&id=4825

Free surface flow towards a well in an "infinite" aquifer

$$H^2 = h_1^2 + \frac{Q}{\pi K} \ln \frac{r}{R}$$



where H is the groundwater level at a distance r from the well and h_1 the initial undisturbed groundwater level.

Flow towards systems of wells in an "infinite" aquifer

The superposition principle is used.

a) Confined flow

$$s = -\frac{1}{2\pi K \alpha} \sum_{i=1}^{n} Q_i \cdot \ln \frac{\sqrt{(x - x_i)^2 + (y - y_i)^2}}{R}$$

b) Free surface flow

$$H^2 = h_1^2 + \frac{1}{\pi K} \sum_{i=1}^{n} Q_i \cdot \ln \frac{\sqrt{(x - x_i)^2 + (y - y_i)^2}}{R}$$

Darcy velocity V versus “real” velocity V_a

$$V_a = \frac{V}{n}$$

Velocity at any point of an infinite confined aquifer with a system of n wells

$$V_x = -\frac{1}{2\pi\alpha}\sum_{i=1}^{n} Q_i \frac{x - x_i}{(x - x_i)^2 + (y - y_i)^2}$$

$$V_y = -\frac{1}{2\pi\alpha}\sum_{i=1}^{n} Q_i \frac{y - y_i}{(x - x_i)^2 + (y - y_i)^2}$$

Theoretical exercise

Ground water is pumped through well A at a constant rate equal to Q, from an "infinite", confined, homogeneous and isotropic aquifer, with constant width a and hydraulic conductivity K. At a point B, which is found at distance r from the well, flow velocity is equal to V_B. If the well flow rate were equal to 2Q and the hydraulic conductivity equal to 3K, what would be the value of water velocity V_{Bnew} at point B (with respect to V_B)?

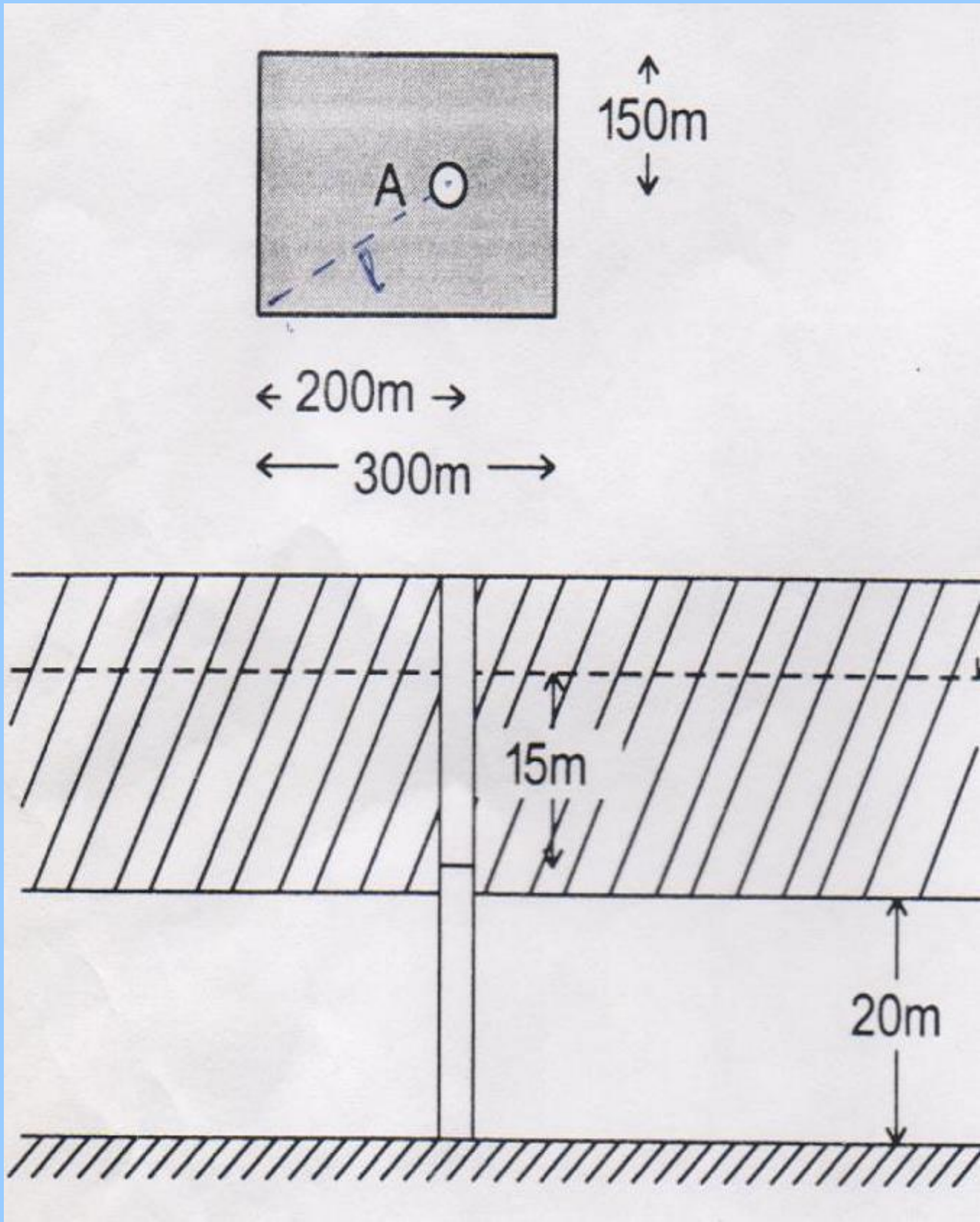
Exercise 1

An accident resulted in the creation of a contamination plume of approximately square shape in the confined aquifer of the following figure. A constant flow rate is pumped from well A, in order to remove the contamination plume.

a) Calculate the time interval which is needed for the complete removal of the contamination plume.

b) What is the percentage of clean water, with respect to the total volume that will be pumped until the complete removal of the plume.

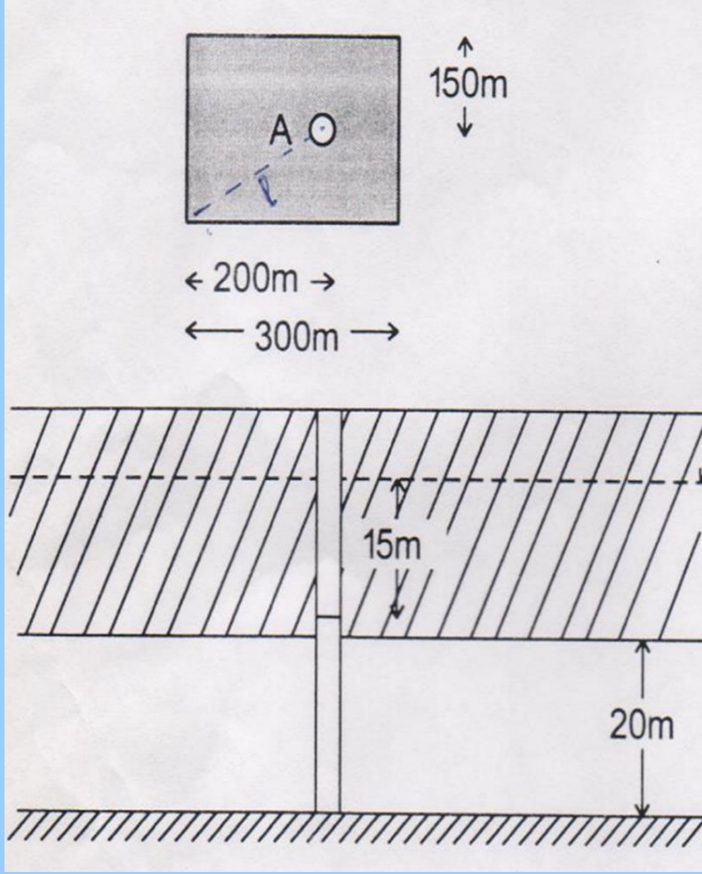
Aquifer's hydraulic conductivity of is K = 5 10^{-5} m/s, its width α = 20m, the effective porosity n = 0.2, the radius of the well r = 0.25 m and the radius of influence R = 1500 m.



Solution

The hydraulic head level drawdown is given as:

$$s = -\frac{Q}{2\pi K \alpha} \ln \frac{r}{R}$$



But s = 15 m at the well (for r = 0.25 m).
It follows that
$Q = 0.0108\ m^3/s$

The distance *l* between the well and the most distant point of the plume is equal to 250 m, from the Pythagorean theorem.

The water volume V, contained in the cylinder of radius *l* and center at the well, should be pumped.

$$V = \pi \cdot l^2 \cdot \alpha$$

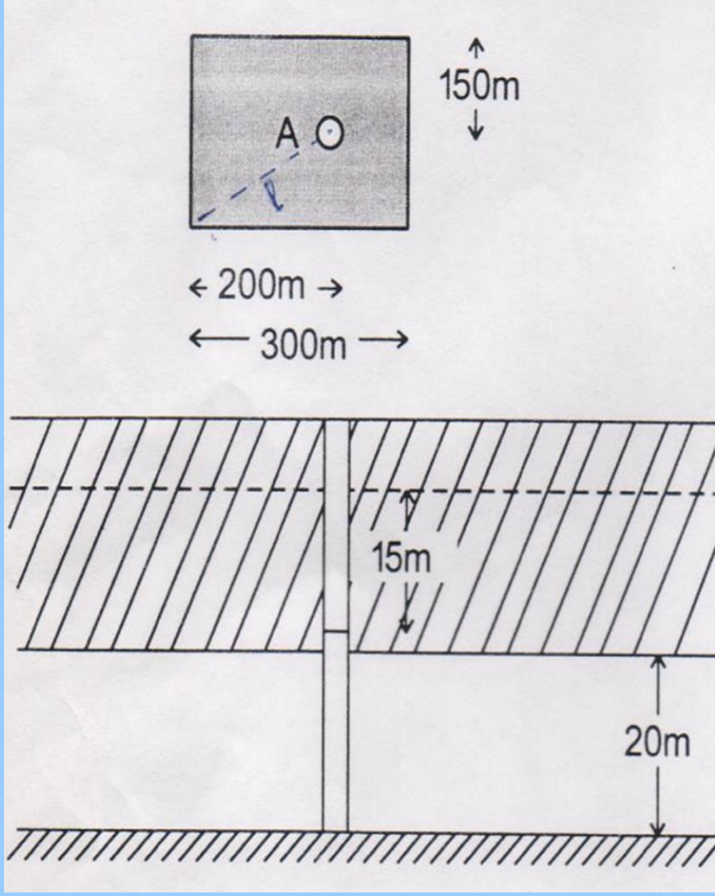
Wrong!

$V = \pi \cdot l^2 \cdot \alpha \cdot n = 785398\ m^3$

Then

$t = V/Q = 72722052\ s = 841.7\ days$

The clean water volume V_1 that has been pumped, is given as:



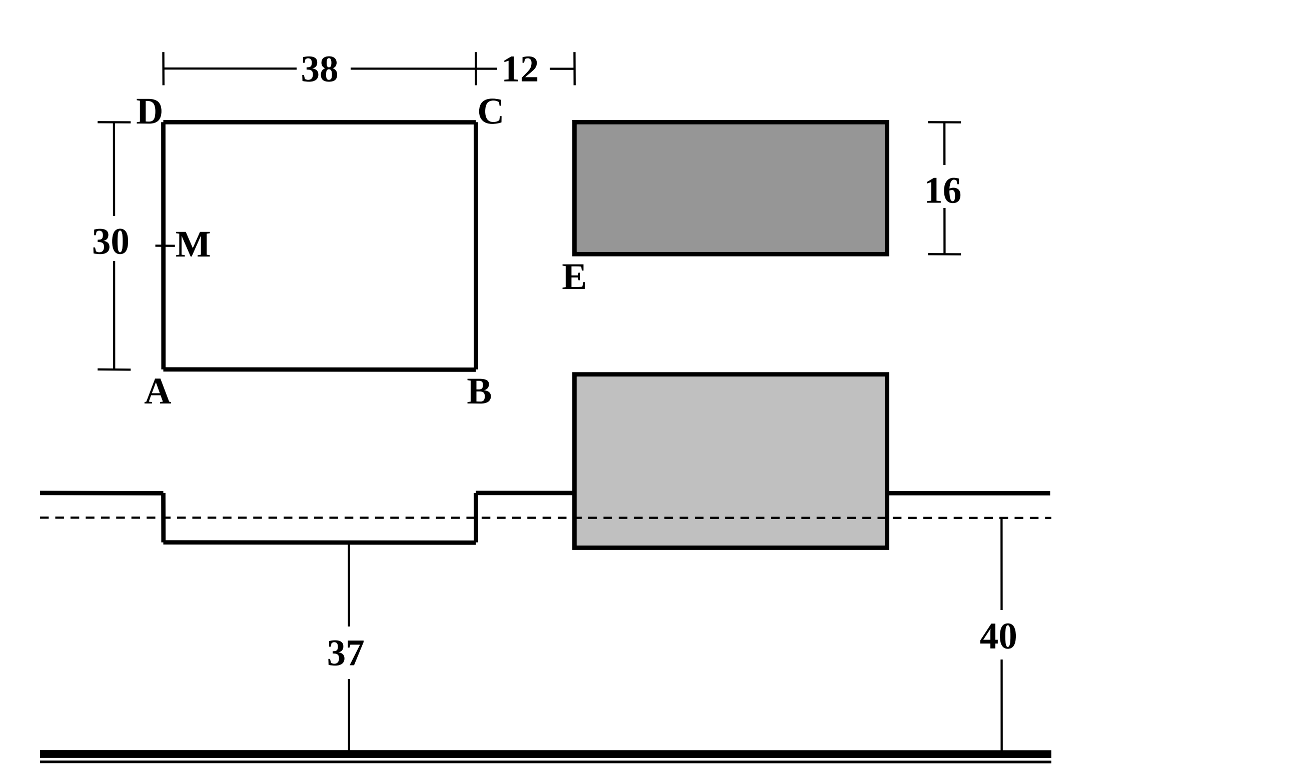
$V_1 = V - 300^2 \cdot 20 \cdot 0.2 = 785398 - 360000 = 425398\ m^3$

The clean water percentage is:

$P_k = V_1/V = 0.5416 = 54.16\%$

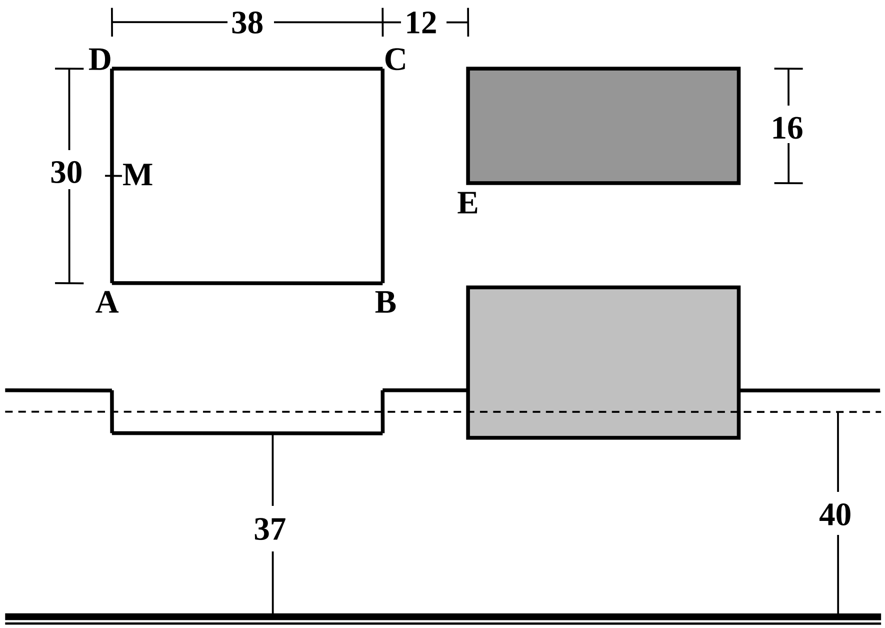
Exercise 2: In order to construct the foundation of a building in the rectangular plot ABCD, shown schematically in the figure, ground water will be pumped from a well, which will be constructed at a point of the perimeter of the excavation. a) At which point are you going to construct the well, in order to minimize the water level drawdown below the adjacent building? b) What is the minimum required well flow rate in order to keep the excavation dry? c) What is the maximum water level drawdown below the adjacent building, because of pumping the aforementioned flow rate?

Data: a) Aquifer's hydraulic conductivity $K = 10^{-4}$ m/s b) Radius of influence of wells $R = 2200$ m c) The aquifer can be considered as infinite.



Solution

A reasonable choice is to place the well at vertex A, since it has the largest distance from the adjacent building. Then, in order to define the required Q the crucial excavation point is C.



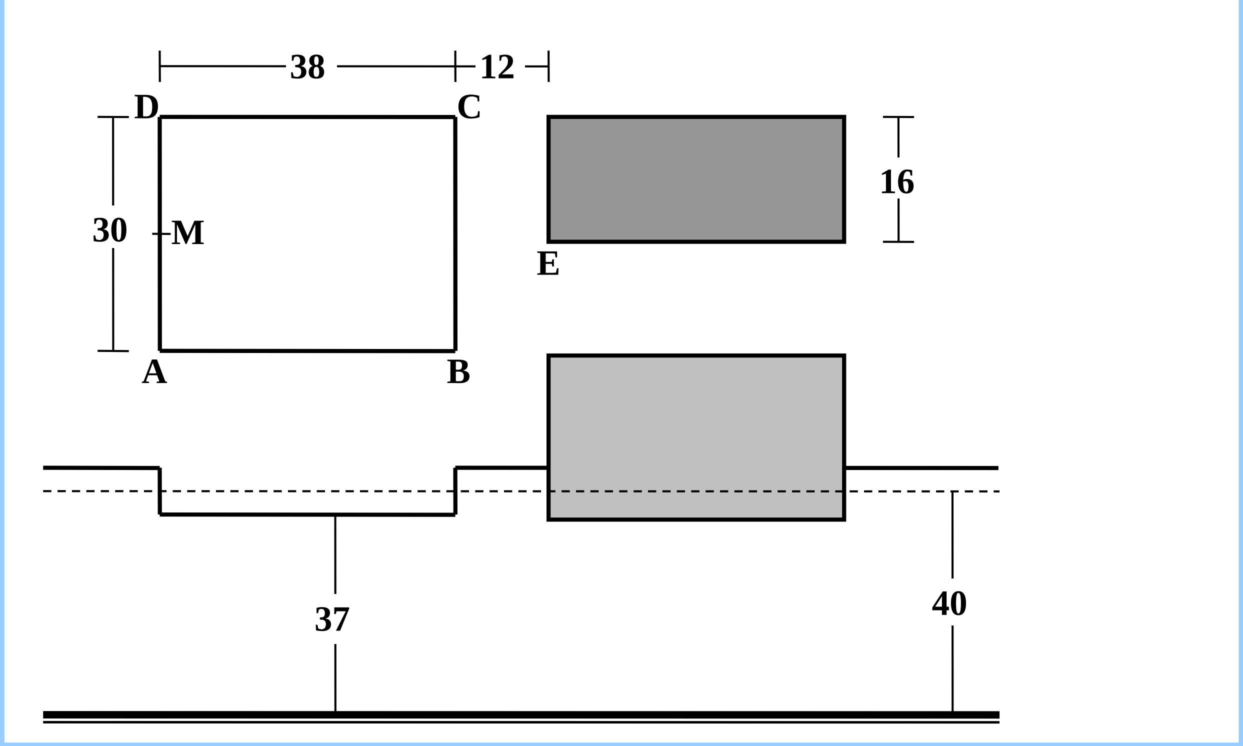
$$H^2 = h_1^2 + \frac{Q}{\pi K} \ln \frac{r_{AC}}{R} \Rightarrow Q = 0.019 \quad m^3/s$$

The largest water level drawdown below the adjacent building appears at point E.

$$H^2 = h_1^2 + \frac{Q}{\pi K} \ln \frac{r_{AE}}{R} \Rightarrow H = 37.06 \quad m \Rightarrow s = 2.94 \quad m$$

Is there any other choice, that could be better?

Placing the well at M, namely the middle of AD, the required well flow rate will be minimized.



$$H^2 = h_1^2 + \frac{Q}{\pi K} \ln \frac{r_{MC}}{R} \Rightarrow Q = 0.0182 \quad m^3/s$$

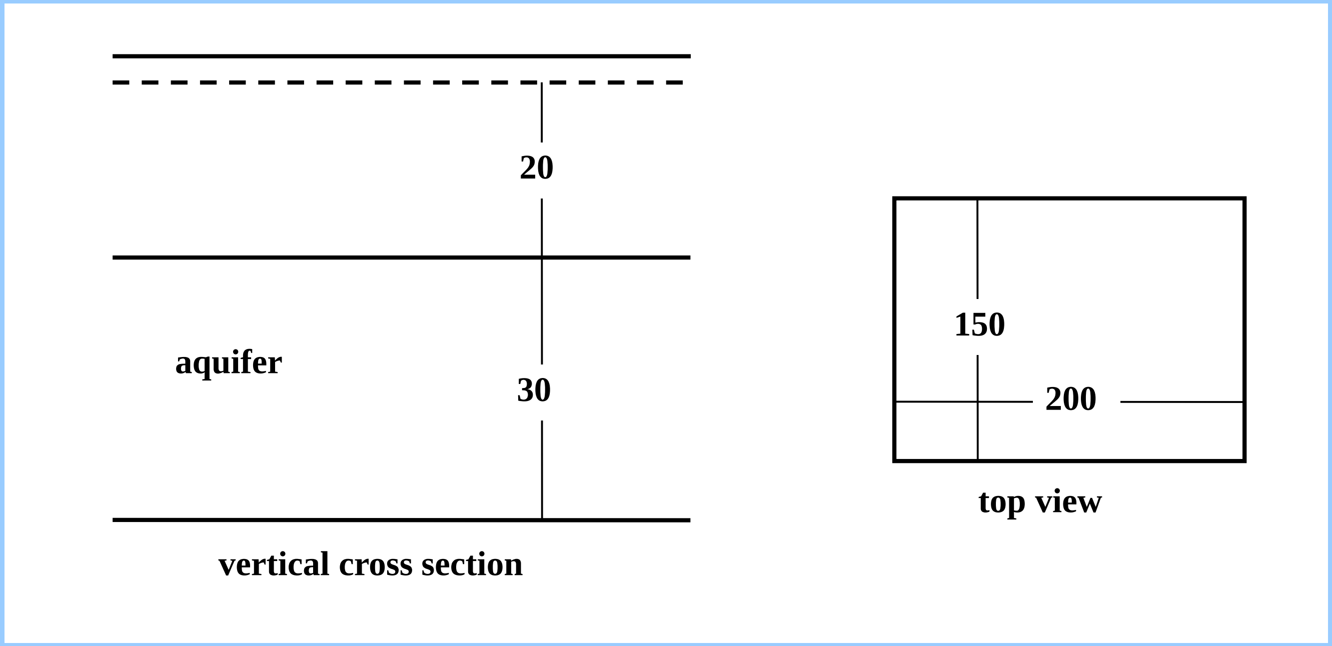
The largest water level drawdown below the adjacent building appears at the point closest to M (at a distance of 50 m).

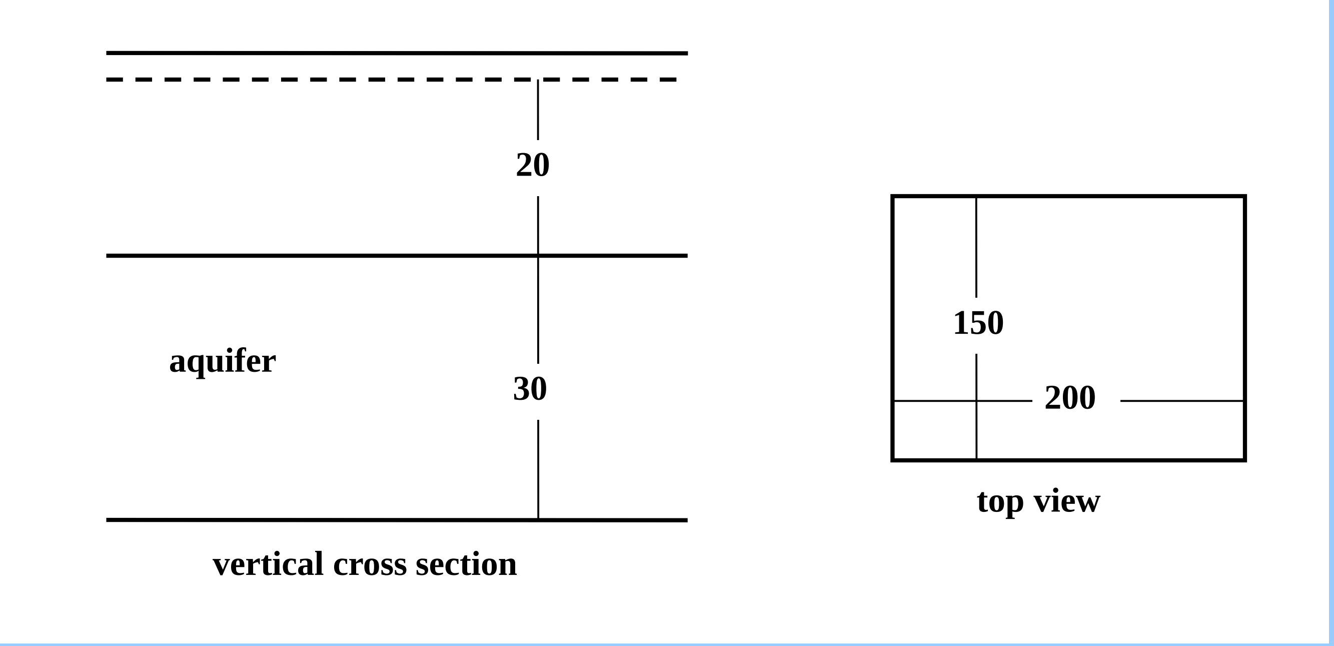
$$H^2 = 40^2 + \frac{0.0182}{\pi 10^{-4}} \ln \frac{50}{2200} \Rightarrow H = 37.16 \quad m \Rightarrow s = 2.84 \quad m$$

Exercise 3

The owner of field A needs to pump groundwater at a flow rate $Q = 0.028\ m^3/s$ from the underlying "infinite" confined aquifer. Find the minimum number of wells that should be constructed inside field A, in order to pump the required flow rate without creating free surface flow conditions at any point of the aquifer.

Data: a) Aquifer's hydraulic conductivity $K = 4 \cdot 10^{-5}$ m/s b) Aquifer's width α = 30 m c) Radius of influence of the wells R = 1500 m d) Well radius r_0 = 0.3 m e) Initial water level H = 50 m f) Field dimensions: 200 x 150 m.





<u>Solution</u>

For 1 well only:

$$s = -\frac{Q}{2\pi K\alpha}\ln\frac{r_0}{R} = 31.64\ \text{m} > 20\ \text{m}$$

For 2 wells, as far as possible from each other:

$$L = \sqrt{200^2 + 150^2} = 250\ \text{m}$$

$$s = -\frac{Q}{4\pi K\alpha}\ln\frac{r_0}{R} - \frac{Q}{4\pi K\alpha}\ln\frac{L}{R} = 15.82 + 3.33 = 19.15\ \text{m} < 20\ \text{m}$$

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End of Unit

Thessaloniki,

December 2016